Amortized Weighted Averaging for Multimedia Broadcast and Multicast Systems

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***Abstract*—We consider the inefficiencies of conventional multimedia broadcast multicast systems (CVM) limited by the user with worst signal-to-noise-ratio (SNR) and we present a novel technique based on amortized weighted averaging (AWG) of users' SNR. We model the system using ordered statistics of exponential and uniform distribution and derive statistics for the density functions. We then apply the modelling results to study the performance of wireless multicasting over Rayleigh fading channels. More specifically, we obtain the outage probability and mean capacities for broadcast and multicast systems. Finally we compare our proposed scheme with the conventional scheme. Numerical results show that proposed AWG is reliable, avoids outage, accommodates more users and is more efficient than the conventional multicasting especially in the low SNR region.**

***Index Terms*—Multicasting, outage probability, weighting factor, reliability, mean capacity, multicast, MWF, CVM, AWG.**

I. Introduction

FOR several decades, researchers have assumed that users at the cell edge in a cellular network have higher probability of outage occurrence. This is a common assumption especially for multimedia broadcast multicast systems (MBMS) in modern communication systems where several users should receive the same transmitted signal. Conventional wisdom shows that a chain is as strong as its weakest link, hence, to avoid service outage in MBMS, it is believed that the group transmission rate and quality of services (QoS) should be defined in terms of the rates which the user with the minimum signal-to-noise-ratio (min-SNR) at the cell edge can decode successfully [1]. However, such assumptions that the min-SNR is sufficient for MBMS is not always valid for two major reasons: First, without formulating the problem as a MaxiMin optimization problem the min-SNR cannot enhance the system performance because the user with the min-SNR bottlenecks the system performance [1]. Solving such problem typically lends itself as a constrained optimization which results in higher computation and system design complexity. Secondly, min-SNR approach assumption may prove inefficient especially when satisfactory QoS is required. Consider the case where the minimum spectral efficiency for guaranteed QoS and error-free decoding is R_0 *(bps/Hz)*. The normalized instantaneous capacity for user with worst SNR is R_{min}. It is clear that $R_{min} \geq R_0$ is required for successful transmission, else user experience would degrade considerably. Typically, the min-SNR approach ensures that users in the group will have similar success rates but it is surely not the best solution.

Several existing works have used variations of min-SNR as their underlining approach. In [2], authors proposed a MaxiMin-type optimization technique that attempts to transmit same data on multiple channel pair such that each group receives the *minimum rates* out of possible maximum. But channel pairing will also reduce usable resources by almost 50%. Authors of [3] proposed a group partitioning with MaxiMin-type technique where users are partitioned into groups based on comparable pathloss instead of their data requirements. The *minimum SNRs* in each partition are selected as baselines. However, the approach also requires more signal processing overhead and multiple transmission of same data. Note that the aforementioned schemes use min-SNR as baseline (cf. [1], [2], eq.(10) and [3], eq.(1)). Hence, we categorize their approaches as MaxiMin-based conventional MBMS (CVM) scheme.

Our focus in this paper is to investigate a novel multicast transmission scheme where each user's SNR is weighted with a multicast weighting factor (MWF). The main idea is to offset the detrimental impact of least and highest SNR and then obtain a systematic *amortized*[1] *weighted averaging (AWG)*. We first derive the probability density function (PDF), cumulative distribution function (CDF) and moment generating function (MGF) for broadcast and multicast. We then present an analysis of the much needed outage probability which have been lacking in multicast literature. Thereafter, we compare AWG outage performance with the CVM scheme and show the possible tradeoff. The AWG approach is modest, concise and can significantly improve system performance. Besides, it is applicable to any multicast performance evaluation and can be used instead of (or in conjunction with) the CVM scheme.

II. System Model

In this section, we first provide a system description as well as axioms and definitions of the AWG scheme. Thereafter, we present the key results on the statistics of our system model.

A. System Description

Consider an OFDM-based cellular network with a base station and user $k \in \kappa_g$ where κ_g is the set of users in multicast group g. Total users in each group is K_g, total number of groups is G and the SNR of each user is denoted as a random variable (r.v.) X. With the assumption of an independent and identically distributed (i.i.d.) multipath Rayleigh fading channel, the PDF of $X_{k,g}$ becomes an i.i.d. exponential r.v.,

Manuscript received February 7, 2014. The associate editor coordinating the review of this letter and approving it for publication was Y.-D. Lin.

This research was supported by the EWRC center, and by Leading Foreign Research Institute Recruitment Program through NRF, Korea, funded by the Ministry of Science, ICT and Future Planning (MSIP) (2009-00422).

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Digital Object Identifier 10.1109/LCOMM.2014.033114.140274

[1]monotonic reduction in value through systematic spreading.

SNR	X_1	...	X_r	...	X_{K_g-1}	X_{K_g}	Ascen.

$\otimes$

WMF	Y_{K_g}	...	$Y_{\tilde{r}=K_g-r+1}$	...	Y_2	Y_1	Descen.

Fig. 1. Schematic of the MWF. r and $\tilde{r}$ are the post-sort index.

$X_{k,g} \sim Exp(\lambda_g)$ where λ_g is the average SNR of group g. For notational convenience, we assume when $G = 1$ we have a broadcast system, consequently, we can omit subscript g such that $X_{k,g} = X_k$ and λ_g is simply λ. Moreover, for $G > 1$, we assume a frame-based system in which decisions on multicast transmission rates are made at the beginning of each time slot and a user who cannot correctly decode received data may have better channel gain in the next frame transmission [2].

Axiom 1 (The Multicast Weighting Factor (MWF)). *Let X_r be users' SNR arranged in ascending order, r being the post-sort index of X_k. Let $Y_{\tilde{r}}$ denote a positive-value stochastic weighting coefficient sorted in descending order as shown in Fig. 1. A user's weighted SNR is then defined as:*

$$X_r \cdot Y_{K_g-r+1} \quad for \quad r = 1, 2, ..., K_g, \tag{1}$$

From eq. (1), we note that for special cases where $Y_{\tilde{r}} = 1$, the min-SNR X_1 is unaffected and when $Y_{\tilde{r}} = 0$, the highest SNR X_{K_g} is treated as outlier; which make sense in multicasting.

Definition 1 (Amortized Weighted averaGing (AWG), $\boldsymbol{H}$). *Given a multicast group with K_g users, let $\tilde{\mathbf{X}} = X_1, ...X_r, ...X_{K_g}$ be users' SNR and $Y_1, ..., Y_{\tilde{r}}, ..., Y_{K_g}$ be the MWF. Then, the AWG-based SNR of the group is defined as:*

$$H = \frac{1}{K_g}\sum_{r=1}^{K_g}[Z_r = Y_{\tilde{r}} \times X_r]. \tag{2}$$

It is clear that to obtain the PDF of H, f_H, we should obtain the PDF f_{Z_r} for the r.v. Z_r which is the product of two i.i.d. r.vs $X_r \sim Exp(\lambda_g)$ and $Y_{\tilde{r}}$. Note that the derivation of density f_{Z_r} is not straightforward due to the orderedness of the random variates. While there may be other ways to denote the weighting coefficient $Y_{\tilde{r}}$, but for the purpose of this work, without loss of generality, we model $Y_{\tilde{r}}$ as a continuous standard uniform r.v. with $Y_{\tilde{r}} \sim Unif[0, 1]$ for $\tilde{r} = K_g - r + 1$.

Lemma 1 (PDF of X_r and $Y_{\tilde{r}}$). *For set of i.i.d. r.v. $\tilde{\mathbf{X}}$, the order of SNR of all users can be represented as an ordered statistics: $X_{(1)}, ..., X_{(\tau)}, ..., X_{(K_g)}$, where $X_{(\tau)}$ is the τ-th order of $X_{k\in\kappa_g}$. $X_{(1)} = \min[X_1, ..., X_k, ...X_{K_g}]$ while $f_{X_{(1)}}$ denotes the PDF of the min-SNR. For any τ, $f_{X_{(\tau)}}$ is:*

$$f_{X_{(\tau)}}(x) = \frac{n! f(x) F(x)^{\tau-1}(1 - F(x))^{n-\tau}}{(\tau - 1)!(n - \tau)!}, \tag{3}$$

Hence, $f_{X_{(r)}}(x)$ and $f_{Y_{(r)}}(y)$ are computed as:

$$f_{X_{(r)}}(x) = \frac{Q_r}{\lambda} exp[-\frac{x(K_g + 1 - r)}{\lambda}] \left(1 - e^{-\frac{x}{\lambda}}\right)^{r-1}, \tag{4}$$

$$f_{Y_{(r)}}(y) = Q_r(1 - y)^{r-1} y^{K_g-r}, \tag{5}$$

where $Q_r = \frac{K_g!}{\Gamma(r)\Gamma(K_g-r+1)}$. Note that the ordered statistics are dependent and non-identical.

Theorem 1 (PDF of $Z_{(r)}$). *Let $X_{(r)} \sim f_{X_{(r)}}$ and $Y_{(r)} \sim f_{Y_{(r)}}$ be two random variables that are independent of each other. The PDF $f_{Z_{(r)}}$, for $Z_{(r)} = X_{(r)} \cdot Y_{(\tilde{r})}$ is given as:*

$$f_{Z_{(r)}}(z) = \sum_{m=0}^{r-1}\sum_{n=0}^{r-1} A \left(\frac{\lambda}{wz}\right)^{-v} \Gamma\left(-v, \frac{wz}{\lambda}\right) \tag{6}$$

where $\underset{m,n,r}{A} = \frac{(K_g!)^2(-1)^{m+n}\binom{r-1}{m}\binom{r-1}{n}}{\lambda(\Gamma(r))^2(\Gamma(K_g-r+1))^2}$, $\underset{m,r}{v} = (K_g + m - r)$ and $\underset{n,r}{w} = (K_g + n - r + 1)$. $\Gamma(\cdot)$ is the incomplete Gamma function; m and n are the indices of the binomial expansions.

***Proof*: *PDF of the product of two continuous r.v. is given as $f_Z = \int_{-\infty}^{\infty} \frac{1}{|y|} f_{X,Y}\left(\frac{z}{y}, y\right) dy$, where $f_{X,Y}$ is the joint PDF.*

$$f_{Z_{(r)}}(z) = \int_0^1 \frac{Q_r^2(1-y)^{r-1} y^{K_g-r}\left(1-e^{-\frac{z}{\lambda y}}\right)^{r-1} e^{-\frac{z\theta_r}{\lambda y}}}{\lambda y} dy, \tag{7}$$

where $\theta_r = (K_g - r + 1)$. Using binomial expansion on both $(\cdot)^{r-1}$, rearranging the integral, collecting the sums from the binomials, gathering the independent coefficients and merging the powers of their exponents, we finally have the form:

$$f_{Z_{(r)}}(z) = \sum_{m=0}^{r-1}\sum_{n=0}^{r-1}\int_0^1 B y^{v-1} e^{-\frac{z(n+\theta)}{\lambda} y^{-1}} dy, \tag{8}$$

where $\underset{m,n,r}{B} = \frac{Q^2(-1)^{m+n}\binom{r-1}{m}\binom{r-1}{n}}{\lambda}$. We can see that the integral part of (8) reduces to the form eqn. (3.381:8) in [4]:

$$\int_0^u y^\alpha e^{-\beta y^\mu} dy = \frac{\Gamma(\xi, \beta u^\mu)}{\mu\beta^\xi}, \tag{9}$$

where $\xi = \frac{\alpha+1}{\mu}$, $u = 1$ and $\mu = 1$. When appropriate value from (8) are substituted into (9) above and further simplified, we obtain the result in (6) which we plotted in Fig. 2(a). ■

Corollary 1 (MGF of AWG-SNR $H = \frac{1}{K_g}\sum_{r=1}^{K_g} Z_r$). *Let $Z_1, ..., Z_{(r)}, ...Z_{K_g}$ be weighted SNR of users in group g for $Z_{(r)} \sim f_{Z_{(r)}}$. Let the MGF of $M_{Z_{(r)}}(-s) = E[e^{-sz}]$. Then, $M_H(-s) = \prod_{r=1}^{K_g} M_{Z_{(r)}}(-\frac{s}{K_g})$ is calculated as:*

$$M_H(-s) = \prod_{r=1}^{K_g}\sum_{m=0}^{r-1}\sum_{n=0}^{r-1} P\, {}_2F_1(1, v + 1; v + 2; -u), \tag{10}$$

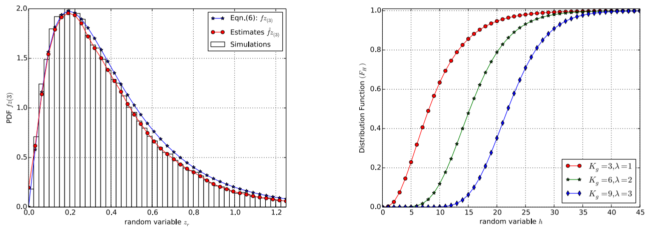
where $\underset{m,n,r}{P} = \frac{(K_g!)^2(-1)^{m+n}\binom{r-1}{m}\binom{r-1}{n}}{(v+1)w(\Gamma(r))^2(\Gamma(K_g-r+1))^2}$ and $u = \frac{\lambda s}{K_g w}$. Symbol ${}_2F_1(\cdot;\cdot;\cdot;\cdot)$ is the Gauss hypergeometric series [4].

Proof: Taking products of $E[e^{-\frac{s}{K_g}z}]$ for $r = 1, ..., K_g$ and using the linearity of integral and sum; the integral in (8) can be evaluated with the help of [4], which once used with some extra manipulations leads to the desired result in eqn. (10).

Corollary 2 (The CDF of H - $F_H(h)$). *Given the MGF of H, $M_H(-s)$, the CDF of H is given by the differential property of Laplace Transform, $F_H(h)$ and denoted as:*

$$F_H(h) = \mathcal{L}^{-1}[\frac{M_H(-s)}{s}], \tag{11}$$

where $\mathcal{L}^{-1}[\cdot]$ denotes the familiar inverse Laplace transform. Substituting (10) into (11) and solving gives the CDF $F_H(h)$. Since the distribution of a r.v. is completely characterized by its CDF, we show the numerical plot of $F_H(h)$ in



(a) $f_{Z_{(3)}}$ (6) vs. Monte Carlo Sim. (b) CDF $F_H(h)$ (11).

Fig. 2. As a double check, the derived PDF $f_{Z_{(r)}}$ in (6) was validated with Monte Carlo simulation in 2(a) for $r = 3$, $K_g = 5$ and $\lambda = 1$ while 2(b) shows the CDF $F_H(h)$ for $K_g = \{3, 6, 9\}$ and average SNR $\lambda = \{1, 2, 3\}$.

Fig. 2(b) by using a multi-precision Laplace inversion method [5].

III. Performance Analysis of AWG

In this section we analyze the performance of AWG in terms of outage probability,$P^W_{out}(R_0)$ and mean capacity. $P^W_{out}(R_0)$ measures the probability that *at least one* of the users in the multicast group cannot satisfy the minimum spectral efficiency R_0 which is the threshold required for guaranteed QoS. On the other hand, reliability function $R^W_{el}(R_0)$ is the probability that outage will not occur and the system will continue to perform beyond a given threshold R_0. Both metrics are related by:

$$P^W_{out}(R_0) + R^W_{el}(R_0) = 1. \tag{12}$$

A. Outage & Reliability for Multicast Systems Using AWG

Suppose there are G multicast groups in the system and to maximize the system capacity, the multicast group with the highest SNR is scheduled for transmission.The CDF $F_{U^\star}$ where $U^\star = \max[U_1, U_2, ..., U_G]$ can be easily determined. Given the CDF $F_U(u) = Pr(U_g \leq u)$ of an arbitrary group g, it is well-known that the CDF of the maximum $F_{U^\star}(u)$ of G independent r.v is the products of the individual $F_U(u)$:

$$F_{U^\star}(u) = (F_U(u))^G = \prod_{g=1}^{G} F_U(u), \tag{13}$$

Thus, outage probability in multicast systems where the group with maximum SNR is scheduled for transmission is given as:

$$P^W_{out}(R_0) = \prod_{g=1}^{G} 1 - \left(1 - F_U(2^{R_0} - 1)\right)^{K_g}. \tag{14}$$

When $F_H(h)$ and $F_X(x)$ are substituted for $F_U(u)$ we have the outage for AWG and CVM respectively. Following from (14), $R^W_{el}(R_0)$ can be computed using relationship in (12). It is important to note that broadcast systems are special cases of multicast where $G = 1$. Hence, broadcast outage can be represented as ${}^bP^W_{out}(R_0) = 1 - (Pr[C \geq R_0])^K$:

$${}^bP^W_{out}(R_0) = 1 - \left(1 - F_H\left(2^{R_0} - 1\right)\right)^K. \tag{15}$$

$C = Log_2(1 + x)$ is the normalized capacity, $K = K_{(g=1)}$ is the group users. For CVM, we replace $F_H(\cdot)$ with $F_{X_{(1)}}$ which can be determined by integrating Lemma (1), eqn. (4).

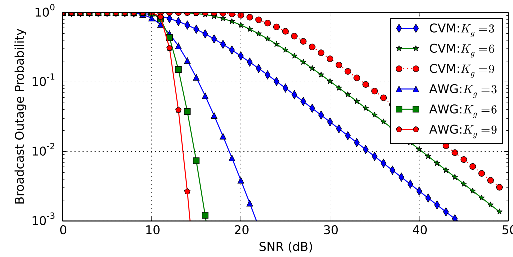


Fig. 3. Single group outage probability with minimum required $R_0 = 2.$.

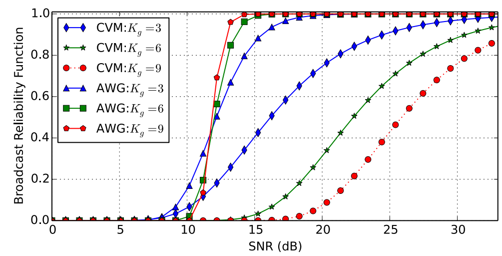


Fig. 4. Reliability function for single group multicast with $R_0 = 2.$.

B. Mean System Capacity

Mean capacity for AWG broadcast group with K users [1]:

$${}^b_W C(K) = E(KC) = K \int_0^\infty log_2(1 + h)\hat{f}_H(h)dh. \tag{16}$$

where $\hat{f}_H(h) \approx f_H(h) = \mathcal{L}^{-1}[M_H(-s)]$ is the interpolated PDF of H derived from the numerically evaluated $f_H(h)$. If we substitute $f_{X_{(1)}}(x)$ for $\hat{f}_H(h)$ in (16), we get the mean broadcast capacity for CVM. Similarly for the AWG multicast capacity ${}^m_W C(K_g, G)$, we follow from (13) and define:

$${}^m_W C(K_g, G) = K_g \int_0^\infty log_2(1 + u) f_{U^\star}(u)du, \tag{17}$$

where $f_{U^\star}(u) = G\left(F_U(u^\star)\right)^{G-1} f_U(u^\star)$ is the PDF of $U^\star$ [2]. If we substitute $\hat{F}_H(h), \hat{f}_H(h)$ and $F_X(x), f_X(x)$ for $F_U(u^\star), f_U(u^\star)$, we obtain results for AWG and CVM.

IV. Numerical Evaluation, Results and Discussions

We perform a numerical evaluation to validate the analysis in section III. Fig. 3 and 4 discuss the broadcast system, Fig. 5 the multicast system while and Fig. 6, 7 explain the tradeoff.

In Fig. 3, we show the outage probability of both CVM and AWG when $R_0 = 2$ and $K_g = (3, 6, 9)$. Observe in Fig. 3 that outage increases as number of users in the group also increases. This behaviour clearly shows that the system will get saturated at some point as K_g grows. But notice that outage begins to drop drastically for the AWG scheme (at group $SNR \simeq 12dB$) as K_g increases while still satisfying R_0 because AWG does not just use the min-SNR but it amortizes the SNR of all users in the group using *Axiom* 1 which compensates for the differences in SNR. Hence, we confirm that *outage in a broadcast system with uniform data rate requirement depends on the number of users in the group.*

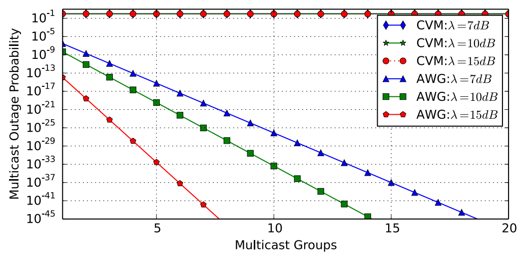


Fig. 5. Multi-group outage with fixed $K_g = 5$ and group SNR $R_0 = 3$.

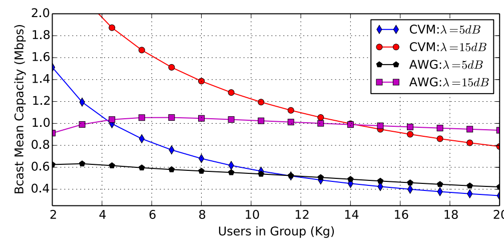


Fig. 6. Broadcast Mean capacity tradeoff for $R_0 = 3$, $\lambda_g = \{5dB, 15dB\}$.

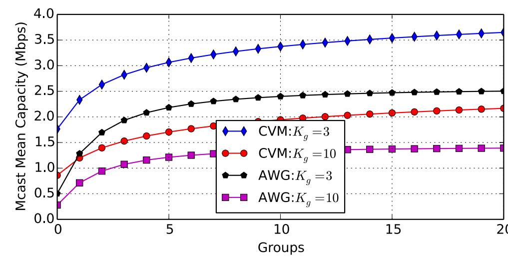


Fig. 7. Multicast Mean capacity tradeoff when $R_0 = 3$, $\lambda_g = 10dB$.

Results for AWG is particularly interesting because it clearly shows that it is possible to avoid system saturation that usually occur in conventional multicasting as users in the group grows.

Fig. 4 shows the system reliability for broadcast system. It is clear that the significance of AWG starts getting noticeable from the mid-SNR region of $15dB$ where AWG is apparently more reliable that the CVM approach. This results evidently strengthens our observation in Fig. 3. However, as the group SNR increases, both CVM and AWG will plainly converge.

Fig. 5 shows the outage performance for a multicast system with up to maximum of 20 groups with average SNR $\lambda_g = \{7, 10, 15\}dB$ and $R_0 = 3$. It shows that as G and R_0 increase, the constraints on the system and competition for system resources become more intense. It is clear that for both schemes, probability of system outage is quite low but especially much lower for the AWG scheme because the group with maximum potential to maximize the system utilization is selected for transmission. Fig. 5 is peculiar because it shows what happens when users' QoS constraints R_0 increases without corresponding increase in the number of users K_g and group SNR λ_g, in which case, outage possibility increases. Following from Fig. 5 where AWG outage is lower, we maintain that the reliability performance for AWG is also significantly better than the CVM. In fact, reliability for AWG $\simeq 1$ due to its very low outage probability compared to CVM.

Fig. 6 shows the effect of group size on the system mean capacity. As K_g increases in CVM at fixed SNR, the system approaches saturation, thus confirming behaviour in Fig. 3. However, in AWG, throughput is almost uniform. *Capacity gaps between the two schemes imply that benefits of AWG become obvious only when there are many users in the group.* While lower throughput is not always desirable from user's perspective, the reduction allows more users to be accommodated in the multicast system without saturation. Similar behaviour is observed for multicast in Fig. 7 causing the gap between both schemes to close up as users per group increase. Note that despite the reduction in AWG compared to CVM, the AWG still satisfy the required minimum QoS $R_0 = 3$. We consider such unique complementary behaviour of AWG as an advantage which may allow both schemes to coexist.

V. Conclusion

In this paper, we have considered the outage probability for the emergent multimedia broadcast and multicast systems (MBMS). We proposed, defined and analyzed a unique amortized weighted averaging (AWG) as an alternative to conventional min-SNR based assumptions in multicasting. We derived the probability distribution, density function as well as the outage probability for both broadcast and multicast systems. Through our analytical results, we show that contrary to popular opinion in conventional multicasting (CVM), *it is possible to systematically exploit the users' channel perception and prevent system saturation as number of users in a multicast group increases, all without complex optimization requirements.* We also showed that AWG is reliable, avoids outage, accommodate more users and more efficient than the CVM especially in the low SNR region. In practical systems, AWG can be implemented as a radio resource management (RRM) submodule, where the transmission rate can be determined before the system resources are allocated.

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